Evaluate States and Radiation of Hydrogen Atom with Anomalous Zeeman Effect

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Abstract: **I study the state structure and radiation of hydrogen atom with anomalous Zeeman Effect. I get hold of Zeeman Hamiltonian from the interaction of magnetic momentum and external magnetic field in matrix form and its determination. Use the approximation of perturbation method; I calculate the energy and wave function of hydrogen atoms. The result indicates that, the spectrum of atom is hyper_ne structure. And obtain two parts of waves which are plane wave and matter wave.**

Keywords: **Hydrogen atom; Zeeman Effect; approximation method; state structure; radiation energy.**

1. INTRODUCTION

Perturbation Theory is an extremely important method which solves the Quantum System will be affected by a small change in the potential formed disturbance. The Zeeman Effect is the splitting of the energy levels of an atom when it is placed in an applied externally magnetic field and without the magnetic field sub-levels have the same energy but with field the degeneration of the levels with different is cancelled and the line is split. The energy shift of atomic states caused by an magnetic field. This shift is due to the coupling of the electron orbital angular momentum to the external magnetic field.

The normal Zeeman Effect occurs when there is no spin magnetic moment states with zero spin are necessary. In singlet systems the spins of the electrons cancel each other. The energy shift of the atomic states in an outer magnetic field can be observed by the wavelength shift of the radiation emitted in atomic transitions between these states. There is not only a magnetic moment of the orbit of an electron state, but also a magnetic moment of the electron spin. This leads to a more complicated behavior of the atomic states in an outer magnetic field. This is called anomalous Zeeman Effect and can be observed in atomic transitions where non-singlet states are involved. The

Splitting occurs because of the interaction of the magnetic moment of the atom with the magnetic field slightly shifts the energy of the atomic levels by an amount.

2. ZEEMAN HAMILTONIAN OPERATOR

In this section we evaluate the Zeeman Hamiltonian operator with the angular momentum. The total Hamiltonian of the hydrogen atom with external strong magnetic field is the algebraic sum of unperturbed and perturbed Hamiltonian.

$$H = H_0 + H_z \qquad (1)$$

The Zeeman Hamiltonian is the scalar product of the magnetic dipole moment and the external magnetic field B.

$$H_z = \int_{90}^{\theta} \tau d\theta \qquad (2)$$

$$\tau = \mu_B \times B \qquad (3)$$

$$\mu \times B = \mu B sin\theta \qquad (4)$$

$$H_z = -\mu_B B \qquad (5)$$

Where

$$\mu_B = \frac{e\hbar}{2m_e c} \qquad (6)$$

is Bohr magneton

$$B = \frac{e}{m_e c r^3}(L + 2S) \qquad (7)$$

$$H_z = \frac{e^2 \hbar}{2m_e^2 c^2 r^3}(L + 2S) \qquad (8)$$

2.1 Orbital Angular Momentum (L)

The orbital angular momentum of a particle is perpendicular to the linear momentum and position of vector r.

$$L = \vec{r} \times \vec{p} \qquad (9)$$

In operator form

$$L = \hat{r} \times \hat{p} \qquad (10)$$

The component of orbital angular momentum in terms of Cartesian coordinate is

$$L = \begin{pmatrix} i & j & k \\ x & y & z \\ p_x & p_y & p_z \end{pmatrix} \qquad (11)$$

$$L_x = yp_z - zp_y$$
$$L_y = zp_x - xp_y$$
$$L_z = xp_y - yp_x \qquad (12)$$

using the commutation relation

$$[L_x, L_y] = i\hbar L_Z$$
$$[L_y, L_z] = i\hbar L_x$$
$$[L_z, L_x] = i\hbar L_y \qquad (13)$$

The commutation relation of Lz and L2 is

$$[L_z, L^2] = 0 \qquad (14)$$

This show that L_z and L^2 commute and two operators have common eigenstate.

let Eigen state of L_z and L^2 denoted by $| \lambda, m \rangle$

$$L_z \mid \lambda, m \rangle = m\hbar \mid \lambda, m \rangle \tag{15}$$

$$L^2 \mid \lambda, m \rangle = \lambda\hbar^2 \mid \lambda, m \rangle \tag{16}$$

In order to determine lamda, we first introduce ladder operator of orbital angular Momentum.

2.1.1 Ladder Operator Of Orbital Angular Momentum

$$L_\pm = L_x \pm iL_y \tag{17}$$

using the commutation relation

$$[L_z, L_\pm] = \pm\hbar L_\pm \tag{18}$$

Employing eq. (18) and operating on the $\mid \lambda, m \rangle$ Eigen state

$$L_z L_\pm \mid \lambda, m \rangle - L_\pm L_z \mid \lambda, m \rangle = \pm\hbar L_\pm \mid \lambda, m \rangle \tag{19}$$

$$L_z L_\pm \mid \lambda, m \rangle = L_\pm L_z \mid \lambda, m \rangle \pm \hbar L_\pm \mid \lambda, m \rangle \tag{20}$$

$$L_z L_\pm \mid \lambda, m \rangle = m\hbar L_\pm L_z \mid \lambda, m \rangle \pm \hbar L_\pm \mid \lambda, m \rangle \tag{21}$$

$$L_z L_\pm \mid \lambda, m \rangle = (m \pm 1)\hbar L_\pm \mid \lambda, m \rangle \tag{22}$$

then,

$$L_+ \mid \lambda.m \rangle = c_+ \mid \lambda.m_{+1} \rangle \tag{23}$$

The inner product of eq. (23)is

$$\langle \lambda, m \mid L_\pm L_\mp \mid \lambda, m \rangle = c_\pm{}^2 \langle \lambda, m_\pm 1 \mid \lambda, m_\pm 1 \rangle \tag{24}$$

Now employing eq. (17)

$$L_\pm L_\mp = L^2 - L_Z{}^2 \pm \hbar L_Z \tag{25}$$

At maximum value m be ℓ

$$L_\pm \mid \lambda, \ell \rangle = 0 \tag{26}$$

Substitute eq. (25) into eq. (24),we have

$$(L^2 - L_Z \pm \hbar L_Z) \mid \lambda, \ell \rangle = 0$$
$$\lambda\hbar^2 - \ell^2\hbar^2 \pm \ell\hbar^2 = 0$$
$$\lambda = \ell(\ell + 1) \tag{27}$$

Then, eq.(16) becomes

$$L^2 \mid \lambda, m \rangle = \ell(\ell + 1)\hbar^2 \mid \lambda, m \rangle \tag{28}$$

using View of eq. (15),eq.(16), eq. (24) becomes

$$c_\pm{}^2 \mid \lambda, m \rangle = (L^2 - L_Z \pm \hbar L_Z) \mid \lambda, m \rangle \tag{29}$$

$$c_\pm{}^2 = \ell(\ell + 1)\hbar^2 - m(m \pm 1)\hbar^2 \tag{30}$$

$$c_\pm = \sqrt{\ell(\ell + 1) - m(m \pm 1)}\hbar \tag{31}$$

$$L_\pm \mid \lambda, m \rangle = \sqrt{\ell(\ell + 1) - m(m \pm 1)}\hbar \mid \lambda, m \pm 1 \rangle \tag{32}$$

2.1.2 Matrix Operators Of Orbital Angular Momentum

The matrix operators for the orbital angular momentum operators Lx, Ly, and

Lz at n=2,l = 1

$$L_z = \begin{pmatrix} \langle \ell,m \mid L_z \mid \ell,m \rangle & \langle \ell,m \mid L_z \mid \ell,m \rangle & \langle \ell,m \mid L_z \mid \ell,m \rangle \\ \langle \ell,m \mid L_z \mid \ell,m \rangle & \langle \ell,m \mid L_z \mid \ell,m \rangle & \langle \ell,m \mid L_z \mid \ell,m \rangle \\ \langle \ell,m \mid L_z \mid \ell,m \rangle & \langle \ell,m \mid L_z \mid \ell,m \rangle & \langle \ell,m \mid L_z \mid \ell,m \rangle \end{pmatrix} \quad (33)$$

$$L_z = \begin{pmatrix} \langle \alpha \mid L_z \mid \alpha \rangle & \langle \alpha \mid L_z \mid \beta \rangle & \langle \alpha \mid L_z \mid \gamma \rangle \\ \langle \beta \mid L_z \mid \alpha \rangle & \langle \beta \mid L_z \mid \beta \rangle & \langle \beta \mid L_z \mid \gamma \rangle \\ \langle \gamma \mid L_z \mid \alpha \rangle & \langle \gamma \mid L_z \mid \beta \rangle & \langle \gamma \mid L_z \mid \gamma \rangle \end{pmatrix} \quad (34)$$

Where $\mid \alpha \rangle =\mid 1,1 \rangle$, $\mid \beta \rangle =\mid 1,0 \rangle$ and $\mid \gamma \rangle =\mid 1,-1 \rangle$

$$L_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (35)$$

Employing eq. (17), similarly for component L_x and L_y, we have

$$L_x = \frac{1}{2} \begin{pmatrix} \langle \alpha \mid (L_+ + L_-) \mid \alpha \rangle & \langle \alpha \mid (L_+ + L_-) \mid \beta \rangle & \langle \alpha \mid (L_+ + L_-) \mid \gamma \rangle \\ \langle \beta \mid (L_+ + L_-) \mid \alpha \rangle & \langle \beta \mid (L_+ + L_-) \mid \beta \rangle & \langle \beta \mid (L_+ + L_-) \mid \gamma \rangle \\ \langle \gamma \mid (L_+ + L_-) \mid \alpha \rangle & \langle \gamma \mid (L_+ + L_-) \mid \beta \rangle & \langle \gamma \mid (L_+ + L_-) \mid \gamma \rangle \end{pmatrix} \quad (36)$$

$$L_x = \frac{\sqrt{2}\hbar}{2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad (37)$$

$$L_y = \frac{\sqrt{2}\hbar}{2} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad (38)$$

Therefore

$$L = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} + \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (39)$$

$$L = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & 1-i & 0 \\ 1+i & 0 & 1-i \\ 0 & 1+i & -\sqrt{2} \end{pmatrix} \quad (40)$$

2.2 Spin Angular Momentum(S)

Spin angular momentum is an intrinsic (innate) of properties of microscope particle such as electron, proton and photons. Spin angular momentum can be expressed

$$S = s_x e_z + s_y e_y + s_z e_z \quad (41)$$

Using the commutation relation

$$[S_x, S_y] = i\hbar S_Z$$
$$[S_y, S_z] = i\hbar S_x$$
$$[S_z, S_x] = i\hbar S_y \quad (42)$$

The commutation relation of Sz and S^2 is

$$[S_z, S^2] = 0 \tag{43}$$

This show that Sz and S^2 commute, so that Sz and S^2 have common eigen

state. let eigen state of S_z and S^2 denoted by $| \lambda, m \rangle$

$$S_z | \lambda, m \rangle = m\hbar | \lambda, m \rangle \tag{44}$$

$$S^2 mid\lambda, m \rangle = \lambda\hbar^2 | \lambda, m \rangle \tag{45}$$

In order to determine lamda ,we first introduce ladder operator of spin angular momentum.

2.2.1 Ladder Operator Of Spin Angular Momentum

$$S_\pm = S_x \pm iS_y \tag{46}$$

using the commutation relation

$$[S_z, S_\pm] = \pm i\hbar S_\pm \tag{47}$$

The same procedure to the orbital angular momentum, the 3 x3 matrix of spin angular momentum is become,

$$s_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{48}$$

$$s_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{49}$$

$$s_y = \frac{\hbar}{2} \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \tag{50}$$

$$S = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2}(1-i) \\ 0 & 0 & 0 \\ \frac{\sqrt{2}}{2}(1+i) & 0 & -\frac{\sqrt{2}}{2} \end{pmatrix} \tag{51}$$

then the total angular momentum

$$J = L + 2S \tag{52}$$

Employing (51) and(40),we have

$$J = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & (1-i) & \sqrt{2}(1-i) \\ (1+i) & 0 & (1-i) \\ \sqrt{2}(1+i) & (1+i) & -\sqrt{2} \end{pmatrix} \tag{53}$$

Using (??) and (53),we got

$$H_z = \frac{e^2\hbar^2}{2m_e^2 c^2 r^3} \begin{pmatrix} \sqrt{2} & (1-i) & \sqrt{2}(1-i) \\ (1+i) & 0 & (1-i) \\ \sqrt{2}(1+i) & (1+i) & -\sqrt{2} \end{pmatrix} \tag{54}$$

ISSN 2350-1030

International Journal of Recent Research in Physics and Chemical Sciences (IJRRPCS)
Vol. 5, Issue 2, pp: (13-20), Month: October 2018 – March 2019, Available at: www.paperpublications.org

$$H_z = \frac{e^2 \hbar^2}{2m_e^2 c^2 r^3} \begin{pmatrix} \sqrt{2} & (1-i) & \sqrt{2}(1-i) \\ (1+i) & 0 & (1-i) \\ \sqrt{2}(1+i) & (1+i) & -\sqrt{2} \end{pmatrix} \tag{55}$$

$$H_z = \frac{e^2 \hbar^2}{2m_e^2 c^2 r^3} \begin{pmatrix} \sqrt{2} & (1-i) & \sqrt{2}(1-i) \\ (1+i) & 0 & (1-i) \\ \sqrt{2}(1+i) & (1+i) & -\sqrt{2} \end{pmatrix} \tag{56}$$

The determinant of the above matrix is

$$H_z = \frac{A}{r^3}(1-i)ev \tag{57}$$

where $A = \frac{2e^2 \hbar^2}{m^e c^2}$

$$H_z = \frac{A}{r^3}(1-i)ev \tag{58}$$

The total Hamiltonian of hydrogen atom is

$$H = \left(-\frac{13.6 a_0}{r} + \frac{A}{r^3}(1-i) \right) ev \tag{59}$$

3. NON DEGENERATE TIME INDEPENDENT PERTURBATION

In this chapter we calculate the eigenvalue and eigenfunction of the hydrogen atoms by employing the time independent perturbation method. when we applied external magnetic field, the degenerate system removed and form the non degenerate system.

$$\hat{H} \mid \psi_n \rangle = E_n \mid \psi_n \rangle \tag{60}$$

where

$$\mid \psi_n \rangle = \mid \psi_n^{(0)} \rangle + \lambda \mid \psi_n^{(1)} \rangle + \lambda^2 \mid \psi_n^{(2)} \rangle + \dots \tag{61}$$

$$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \dots \tag{62}$$

Corresponding the coefficient of power lambda is

$$\lambda^0 : H_0 \mid \psi_n^{(0)} \rangle = E_n^{(0)} \mid \psi_n^{(0)} \rangle \tag{64}$$

$$\lambda^1 : H_0 \mid \psi_n^{(1)} \rangle + H_z \mid \psi_n^{(0)} \rangle = E_n^{(0)} \mid \psi_n^{(1)} \rangle + E_n^{(1)} \mid \psi_n^{(0)} \rangle \tag{65}$$

$$\lambda^2 : H_0 \mid \psi_n^{(2)} \rangle + H_z \mid \psi_n^{(1)} \rangle = E_n^{(0)} \mid \psi_n^{(2)} \rangle + E_n^{(1)} \mid \psi_n^{(1)} \rangle + E_n^{(2)} \mid \psi_n^{(0)} \rangle \tag{66}$$

Employing (51) and(40),we have

$$J = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & (1-i) & \sqrt{2}(1-i) \\ (1+i) & 0 & (1-i) \\ \sqrt{2}(1+i) & (1+i) & -\sqrt{2} \end{pmatrix} \tag{53}$$

Using (??) and (53),we got

$$H_z = \frac{e^2 \hbar^2}{2m_e^2 c^2 r^3} \begin{pmatrix} \sqrt{2} & (1-i) & \sqrt{2}(1-i) \\ (1+i) & 0 & (1-i) \\ \sqrt{2}(1+i) & (1+i) & -\sqrt{2} \end{pmatrix} \tag{54}$$

3.1. First order correction perturbation

Correction to energy multiply eq. (65) by ground state psi (n)

$$\langle \psi_n^{(0)} \mid H_0 \mid \psi_n^{(1)} \rangle + \langle \psi_n^{(0)} H_z \mid \psi_n^{(0)} \rangle = E_n^{(0)} \langle \psi_n^{(0)} \mid \psi_n^{(1)} \rangle + E_n^{(1)} \langle \psi_n^{(0)} \mid \psi_n^{(0)} \rangle \quad (67)$$

$$\langle \psi_n^{(0)} \mid H_z \mid \psi_n^{(0)} \rangle = E_n^{(1)} \quad (68)$$

Employing the (68), we have

$$E_n^{(1)} = A(1-i)ev\langle \psi_n^{(0)} \mid \frac{1}{r^3} \mid \psi_n^{(0)} \rangle \quad (69)$$

$$E_1^{(1)} = \frac{16\pi}{a_0^3} \langle \psi_1^{(0)} \mid \frac{1}{r} \mid \psi_1^{(0)} \rangle \quad (70)$$

$$E_1^{(1)} = \frac{16\pi}{a_0^4} A(1-i) \quad (71)$$

Correction to wave function Multiply eq. (65) by ground state psi (m)

$$\langle \psi_m^{(0)} \mid H_0 \mid \psi_n^{(1)} \rangle + \langle \psi_m^{(0)} \mid H_z \mid \psi_n^{(0)} \rangle = E_n^{(0)} \langle \psi_m^{(0)} \mid \psi_n^{(1)} \rangle + E_n^{(1)} \langle \psi_m^{(0)} \mid \psi_n^{(0)} \rangle \quad (72)$$

$$(E_n^{(0)} - E_m^{(0)})\langle \psi_m^{(0)} \mid \psi_n^{(1)} \rangle = \langle \psi_m^{(0)} \mid H_z \mid \psi_n^{(0)} \rangle \quad (73)$$

$$\langle \psi_m^{(0)} \mid \psi_n^{(1)} \rangle = \frac{\langle \psi_m^{(0)} H_z \mid \psi_n^{(0)} \rangle}{E_n^{(0)} - E_m^{(0)}} \quad (74)$$

Multiply eq. (??) by conjugate ground state psi (m)

$$\mid \psi_n^{(1)} \rangle = \sum_{m \neq n} \frac{\langle \psi_m^{(0)} \mid H_z \mid \psi_n^{(0)} \rangle}{E_n^{(0)} - E_m^{(0)}} \mid \psi_m^{(0)} \rangle \quad (75)$$

$$\mid \psi_1^{(1)} \rangle = \frac{e^2 \hbar^2 \pi}{m_e^2 c^2 a_0^3}(1-i)\left(0.216|\psi_2^0\rangle + 0.00592|\psi_3^0\rangle + ... \right) \quad (76)$$

3.2. Second order correction perturbation

Correction to energy Multiply eq.(66) by ground state psi (n)

$$\langle \psi_n^{(0)} \mid H_0 \mid \psi_n^{(2)} \rangle + \langle \psi_n^{(0)} \mid H_z \mid \psi_n^{(1)} \rangle = E_n^{(0)} \langle \psi_n^{(0)} \mid \psi_n^{(2)} \rangle + E_n^{(1)} \langle \psi_n^{(0)} \mid \psi_n^{(1)} \rangle + E_n^{(2)} \langle \psi_n^{(0)} \mid \psi_n^{(0)} \rangle \quad (77)$$

$$E_n^{(2)} = \langle \psi_n^{(0)} \mid H_I \mid \psi_n^{(1)} \rangle \quad (78)$$

Employing eq. (75), we have

$$E_n^{(2)} = \sum_{m \neq n} \langle \psi_n^{(0)} \mid H_z \mid \psi_n^{(0)} \rangle \frac{\langle \psi_m^{(0)} \mid H_z \mid \psi_m^{(0)} \rangle}{E_n^{(0)} - E_m^{(0)}} \quad (79)$$

$$E_1^{(2)} = \frac{A^2 \pi^2}{a_0^6}\left(0.7 + 0.00027 + ... \right) \quad (80)$$

Correction to wave function Multiply eq. (66) by ground state psi (n)

$$\langle \psi_m^{(0)} \mid H_0 \mid \psi_n^{(2)} \rangle + \langle \psi_m^{(0)} \mid H_z \mid \psi_n^{(1)} \rangle = E_n^{(0)} \langle \psi_m^{(0)} \mid \psi_n^{(2)} \rangle + E_n^{(1)} \langle \psi_m^{(0)} \mid \psi_n^{(1)} \rangle + E_n^{(2)} \langle \psi_m^{(0)} \mid \psi_n^{(0)} \rangle \tag{81}$$

$$(E_n^{(0)} - E_m^{(0)}) \langle \psi_m^{(0)} \mid \psi_n^{(2)} \rangle = \langle \psi_m^{(0)} H_I \mid \psi_n^{(1)} \rangle \tag{82}$$

$$\langle \psi_m^{(0)} \mid \psi_n^{(2)} \rangle = \frac{\langle \psi_m^{(0)} H_z \mid \psi_n^{(1)} \rangle}{E_n^{(0)} - E_m^{(0)}} \tag{83}$$

Multiply eq. (83) by conjugate of ground state psi (n)

$$\mid \psi_n^{(2)} \rangle = \sum_{m \neq n} \frac{\langle \psi_m^{(0)} H_z \mid \psi_n^{(0)} \rangle}{E_n^{(0)} - E_m^{(0)}} \mid \psi_m^{(1)} \rangle \tag{84}$$

$$\mid \psi_1^{(2)} \rangle = \frac{-2ie^4 \hbar^4 \pi^2}{m^4 c^4 a_0^6} \left(0.04678 |\psi_2^0\rangle + 0.001282 |\psi_3^0\rangle + ... \right) \tag{85}$$

4. CONCLUSION

when the electron of magnetic momentum interacted with external magnetic field; the degenerate system is removed and formed nondegenerate system. The coloumb potential is greater than the zeeman hamiltonian. Zeeman hamiltonian is non hermitian operator(non observable)and hermitian operator. The first order correction perturbation energy exceed the second order correction perturbation energy.The eigenfunction of hydrogen atom has plane wave and matter wave at first order correction and only matter wave at second order correction.This wave function is show that the either antisymmetric or antiparity.In general,the value of zeemam hamiltonian Hz = A/r^3 (1 - i)ev is indicate the hyperfine structure of an interaction between electron magnetic momentum and external magnetic field.

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